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Properties of vector mesons at finite temperature
−effective lagrangian approach−

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Abstract

The properties of ρ-mesons at finite temperature (T) are examined with an effective chiral lagrangian in which vector and axial-vector mesons are included as massive Yang-Mills fields of the chiral symmetry. It is shown that, at T^2 order, the effective mass is not changed but only the mixing effect in vector and axial-vector correlator appears.

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I. INTRODUCTION

It is expected that spontaneously broken chiral symmetry is restored in hadronic matter at very high temperature [1], which has been shown for QCD on lattice [2]. The restoration of the chiral symmetry is characterized by melting out of the quark condensate which is known as the order parameter of the chiral phase transition. The vanishing of the quark condensate with temperature also has been shown from some model calculations [3].

We are interest in the phenomena which arise prior to the chiral transition in hot hadronic matter. Even though the quark condensate is not directly observable the change of the condensate at finite temperature would affect on the properties of hadrons. It is desirable to have a direct connection between the properties of hadrons and those of ground state in order to study the chiral transition in hot matter. Of particular interest are the vector meson properties at finite temperature since model calculations show definite relation between chiral symmetry restoration at finite temperature and vector meson mass [4]. Moreover, the change in the vector meson mass can be observed from the shift of the vector meson peaks in dilepton spectrum from hot matter.

The vector meson properties and dilepton spectrum at finite temperature have been studied in various ways [4–12]. Recently, it was realized by Dey et. al. [6] that in the lowest order of $\epsilon = T^2/6f_\pi^2$, where the pion decay constant $f_\pi = 93$ MeV, there is no change in the vector meson mass and only mixing between vector and axial-vector correlator takes place. Since the isospin mixing effect is obtained based only on the PCAC and current algebra, it has to be satisfied in the low temperature limits of any model calculations as long as the model calculations preserve the same symmetry properties. This constraint is indeed satisfied in the low temperature limit of the calculation using QCD sum rule [7,8] and with the effective chiral lagrangian [12].

In this paper the isospin mixing effect is obtained from an effective lagrangian approach in which vector and axial-vector mesons are included as massive Yang-Mills fields of the chiral group and the photon fields are introduced via vector meson dominance assumption.

The result indicates that the effective masses of vector mesons are not modified in the leading order of temperature, $\mathcal{O}(T^2)$. In section 3, we introduce an extra non-minimal coupling term to reproduce the experimental values of axial-vector meson mass and width. With the new term we cannot get the exact mixing effect in vector and axial-vector correlator at finite temperature but the effective masses of vector mesons are still not changed much at the leading T^2 order.

II. ISOSPIN MIXING AT FINITE TEMPERATURE

We consider an effective chiral lagrangian with vector and axial-vector meson fields which are introduced as massive Yang-Mills fields [13];

$$
\begin{aligned}
\mathcal{L} = & \frac{1}{4} f_\pi^2 \mathrm{Tr}[D_\mu U D^\mu U^\dagger] \\
& -\frac{1}{2}\mathrm{Tr}[F^L_{\mu\nu}F^{L\mu\nu} + F^R_{\mu\nu}F^{R\mu\nu}] + m_0^2 \mathrm{Tr}[A^L_\mu A^{L\mu} + A^R_\mu A^{R\mu}] \\
& -i\xi \mathrm{Tr}[D_\mu U D_\nu U^\dagger F^{L\mu\nu} + D_\mu U^\dagger D_\nu U F^{R\mu\nu}],
\end{aligned} \tag{1}
$$

where U is related to the pseudoscalar fields ϕ by

$$
U = exp\left[\frac{i\sqrt{2}}{f_\pi}\phi\right], \quad \phi = \sum_{a=1}^{3} \phi_a \frac{\tau_a}{\sqrt{2}}, \tag{2}
$$

and $A^L_\mu (A^R_\mu)$ are left(right)-handed vector fields. The covariant derivative acting on U is given by

$$
D_\mu U = \partial_\mu U - igA^L_\mu U - igUA^R_\mu, \tag{3}
$$

and $F^L_{\mu\nu}$ $(F^R_{\mu\nu})$ is the field tensor of left(right)-handed vector fields. The A^L_μ and A^R_μ can be written in terms of vector (V_μ) and axial-vector fields (A_μ) as

$$
A^L_\mu = \frac{1}{2}(V_\mu - A_\mu), \quad A^R_\mu = \frac{1}{2}(V_\mu + A_\mu). \tag{4}
$$

The lagrangian can be diagonalized in the conventional way by the definitions

$$
A_\mu \to A_\mu + \frac{g f_\pi}{\sqrt{2} m_0^2}\left(\partial_\mu \phi - i\frac{g}{2}[V_\mu, \phi]\right),
$$

$$\phi \to Z^{-1}\phi,$$
$$f_\pi \to Z^{-1} f_\pi. \tag{5}$$

In terms of new fields we find

$$\mathcal{L}^{(2)} = \frac{1}{2}\mathrm{Tr}\left(\partial_\mu\phi - i\frac{g}{2}[V_\mu,\phi]\right)^2 - \frac{1}{4}\mathrm{Tr}[F^V_{\mu\nu}F^{V\mu\nu} + F^A_{\mu\nu}F^{A\mu\nu}]$$
$$+\frac{1}{2}m_\rho^2\mathrm{Tr}V_\mu^2 + \frac{1}{2}m_a^2\mathrm{Tr}A_\mu^2, \tag{6}$$

where we use

$$Z^2 = \left[1 - \frac{g^2 f_\pi^2}{2m_\rho^2}\right] = \frac{m_\rho^2}{m_a^2}, \tag{7}$$

and the vector and axial-vector meson mass are given by

$$m_\rho^2 = m_0^2, \qquad m_a^2 = m_0^2/Z^2. \tag{8}$$

When we choose $Z^2 = \frac{1}{2}$ we have the KSRF relation, $m_\rho^2 = g^2 f_\pi^2$, and Weinberg mass relation, $m_a^2 = 2m_\rho^2$.

One can calculate the width of ρ-mesons from the given lagrangian as

$$\Gamma(\rho \to \pi\pi) = \frac{1}{6\pi m_\rho^2}|q_\pi|^3 g_{\rho\pi\pi}^2 \tag{9}$$

with

$$g_{\rho\pi\pi} = \frac{g}{4\sqrt{2}}(3 + 2g\xi) \tag{10}$$

To satisfy the Universality of vector meson coupling $g_{\rho\pi\pi} = g/\sqrt{2}$ [1], we choose $2g\xi = 1$. The coupling constant g can be determined from the experimental value of the ρ-width, $\Gamma^{exp}(\rho \to \pi\pi) = 150$ MeV.

We study the properties of vector mesons at finite temperature with the effective lagrangian. Here we assume that the known hadronic interactions can be extrapolated to

[1] There is a factor $1/\sqrt{2}$ because $V_\mu = \frac{\tau^a}{\sqrt{2}}\rho^a_\mu$, where a is the isospin index

finite temperature and describe the interactions among particles in hot hadronic matter. The properties of vector mesons are modified by the interactions with the particles in the heat bath and this modification can be included in the self-energy. The self-energy is defined by the difference between the inverse of the in-medium propagator $D_{\mu\nu}$ and vacuum propagator $D^0_{\mu\nu}$ [14];

$$\Pi_{\mu\nu} = D^{-1}_{\mu\nu} - D^{-1}_{0\mu\nu}. \tag{11}$$

Since the self-energy of the vector fields is the symmetric and transverse it can be written in terms of the projection tensors as

$$\Pi_{\mu\nu} = GP^T_{\mu\nu} + FP^L_{\mu\nu}, \tag{12}$$

where $P^T_{00} = P^T_{0i} = P^T_{i0} = 0$, $P^T_{ij} = \delta_{ij} - k_i k_j/\vec{\mathrm{k}}^2$ and $P^L_{\mu\nu} = k_\mu k_\nu/k^2 - g_{\mu\nu} - P^T_{\mu\nu}$. The functions F and G is given by

$$\begin{aligned} F(k_0, \vec{\mathrm{k}}) &= \frac{k^2}{\vec{\mathrm{k}}^2}\Pi_{00}(k_0, \vec{\mathrm{k}}), \\ G(k_0, \vec{\mathrm{k}}) &= -\frac{1}{2}[\Pi^\mu_\mu(k_0, \vec{\mathrm{k}}) + F(k_0, \vec{\mathrm{k}})], \end{aligned} \tag{13}$$

where we use $k = (k_0, \vec{\mathrm{k}})$ and $k^2 = k_0^2 - \vec{\mathrm{k}}^2$. The propagator of the vector mesons in the medium can be written as

$$\mathcal{D}_{\mu\nu} = -\frac{P^{\mu\nu}_L}{k^2 - m^2_\rho - F} - \frac{P^{\mu\nu}_T}{k^2 - m^2_\rho - G} - \frac{k^\mu k^\nu}{m^2_\rho k^2}. \tag{14}$$

We consider the corrections that come from interactions with thermal pions. In the chiral limit, $m_\pi = 0$, the interaction with pions generates power corrections, controlled by the expansion parameter $\sim T^2/f^2_\pi$, and the thermal corrections can be obtained in a systematic way. Particles with mass M generate contributions of order $exp(-M/T)$ which are exponentially suppressed compared with the effects from thermal pions. In this respect, it is similar to calculation of the loop corrections in chiral perturbation [15]. The self-energy of vector mesons in hot hadronic matter can be expanded in powers of T^2/f^2_π as

$$\Pi_{\mu\nu}(k_0, k; T) = \Pi^{(1)}_{\mu\nu}(k_0, k; T^2/f^2_\pi) + \Pi^{(2)}_{\mu\nu}(k_0, k; T^4/f^4_\pi) + \cdots. \tag{15}$$

The leading contributions can be obtained from one-loop diagrams in fig. 1. Even in the presence of the a_1-meson propagator in the internal loop we can still make a systematic expansion. At the leading order in temperature the contribution from diagram fig. 1-c can written as

$$\Pi^{(c)}_{\mu\nu}(k_0,\vec{\mathrm{k}}) = \frac{1}{2f_\pi^2}k^2(k^2 g_{\mu\nu} - k_\mu k_\nu)T\sum\int \frac{d^3\vec{\mathrm{p}}}{(2\pi)^3}\frac{1}{p^2}\frac{1}{(p-k)^2-m_a^2}. \tag{16}$$

The full expression is given in appendix. For the axial vector mesons with momentum q, we have $1/(q^2-m_a^2)$ in which the momentum q is given by the momentum of thermal pion, p, and the external vector mesons, k. When we do the integration over thermal pions we get

$$T\sum\int \frac{d^3\vec{\mathrm{p}}}{(2\pi)^3}\frac{1}{p^2}\frac{1}{(p-k)^2-m_a^2} \sim \frac{1}{k^2-m_a^2}\int \frac{|\vec{\mathrm{p}}|d|\vec{\mathrm{p}}|}{e^{\mathrm{p}/T}-1}\left(1+\frac{|\vec{\mathrm{p}}|^2}{k^2-m_a^2}+\cdots\right), \tag{17}$$

where we assume that

$$\frac{|\vec{\mathrm{p}}|^2}{k^2-m_a^2} \sim \frac{|\vec{\mathrm{p}}|^2}{m_\rho^2} < 1 \tag{18}$$

With $m_\rho^2 = g^2 f_\pi^2$ we can still expand the correction in powers of T^2/f_π^2.

At the leading order in T^2 we have

$$\begin{aligned} F^\pi &= \frac{k^2}{\vec{\mathrm{k}}^2}\left[g^2 f_\pi^2 \frac{k_0^2}{k^2}\frac{\epsilon}{2} - g^2 f_\pi^2\left(1+\frac{\vec{\mathrm{k}}^2}{2m_\rho^2}\right)\frac{\epsilon}{2}\right], \\ F^{a_1} &= k^2\left(1+\frac{m_a^2}{k^2-m_a^2}\right)\frac{\epsilon}{4}, \end{aligned} \tag{19}$$

where F^π is the contribution comes from pion loops in figs. 1-a,b and F^{a_1} is from $\pi - a_1$ loop in fig. 1-c. The first two terms in F^π are the same terms obtained from the effective model only with charged pions and neutral vector mesons [9]. With including of the a_1 mesons we have F^{a_1} and the last term in F^π. When we use the relations

$$g^2 f_\pi^2 = m_\rho^2, \qquad m_a^2 = 2m_\rho^2, \tag{20}$$

there is an exact cancellation between the last term in F^π and the first term in F^{a_1} and finally we have

$$F = \left(m_\rho^2 + \frac{m_\rho^4}{k^2-m_a^2}\right)\epsilon. \tag{21}$$

The longitudinal mode of vector meson propagator is modified at finite temperature and shows isospin mixing effect in the leading order of temperature;

$$\begin{aligned}\frac{1}{k^2-m_\rho^2-F} &= \frac{1}{k^2-m_\rho^2}+\frac{1}{k^2-m_\rho^2}F\frac{1}{k^2-m_\rho^2}+\cdots \\ &= (1-\epsilon)\frac{1}{k^2-m_\rho^2}+\epsilon\frac{1}{k^2-m_a^2}+\mathcal{O}(T^4)\end{aligned} \tag{22}$$

For transverse mode we have

$$G = g^2 f_\pi^2\frac{\epsilon}{2} - g^2 f_\pi^2\frac{k^2}{2m_\rho^2}\frac{\epsilon}{2} + k^2\left(1+\frac{m_a^2}{k^2-m_a^2}\right)\frac{\epsilon}{4}, \tag{23}$$

which is the same as the longitudinal component. The transverse and longitudinal components are the same at the T^2 order and the mixing effect appears in both modes of vector propagator at finite temperature.

III. EFFECTIVE MASSES OF VECTOR MESONS

Even though the effective lagrangian we used satisfies the Universality and KSRF relations, the lagrangian could not reproduce the experimental values of the masses and decay widths of a_1-mesons. For given parameters with $2g\xi = 1$ and $Z^2 = 1/2$, we have $m_a = \sqrt{2}m_\rho = 1089$ MeV and $\Gamma(a_1 \to \pi\rho) = 53$ MeV while experiments show that $m_a^{\rm exp} = 1260$ MeV and $\Gamma^{\rm exp}(a_1 \to \pi\rho) = 400$ MeV. By adding an extra non-minimal coupling term as [16]

$$\mathcal{L}_\sigma = \sigma {\rm Tr}[F^L_{\mu\nu} U F^{R\mu\nu} U^\dagger]. \tag{24}$$

we can well describe the masses and widths of vector and axial vector mesons with parameters; $g = 10.3063$, $\sigma = 0.3405$, $\xi = 0.4473$ [10]. However, the lagrangian does not satisfy Universality, $g_{\rho\pi\pi} = 3.06(g/\sqrt{2})$ and KSRF relation, $m_\rho^2 = 1.63 g^2 f_\pi^2$.

With these parameters we have

$$F = g^2 f_\pi^2\frac{\epsilon}{2} + k^2\left(2\eta_2^2 - \frac{\lambda g^2 f_\pi^2}{m_\rho^2}\right)\frac{\epsilon}{2} + \eta_2^2 k^2\frac{m_a^2}{k^2-m_a^2}\epsilon, \tag{25}$$

where η_2 and λ are given in the appendix. The second term in F is not canceled and the first and the last term have different coefficients. By introducing an extra term, there is not the same mixing in the vector and axial-vector correlator as shown from the calculation based on current algebra and PCAC, and the effective masses of the ρ-mesons have T^2 dependent corrections.

We obtain the effective mass from the pole position of the propagator at zero three-momentum. Since there is no distinction between the longitudinal and transverse modes in the limit $\vec{\mathrm{k}} \rightarrow 0$, the effective masses of vector mesons are obtained from the equation

$$k_0^2 - m_\rho^2 - F(k_0, \vec{\mathrm{k}} \rightarrow 0) = 0. \tag{26}$$

From eq. (25) and (26) we see that the effective ρ-meson masses are not changed with temperature for $\sigma = 0$, $2g\xi = 1$ and $Z^2 = 1/2$. For $\sigma \neq 0$, we show the temperature dependence of the ρ meson masses in fig. 2. The dashed line is the result obtained from the caluction only with pions and rho-mesons, and the solid line is that obtained with including a_1-mesons in the effective lagrangian. The effective masses of vector mesons are still not changed much with temperature at T^2 order [17]. The increase due to the pion loops is almost canceled out when we include a_1 mesons.

IV. CONCLUSION

We study the properties of vector mesons at finite temperature with an effective chiral lagrangian in which the vector mesons are introduced as massive Yang-Mills fields. It is shown that, at the leading order of temperature, the isospin mixing effect in vector and axial-vector correlator take places and the effective masses of vector mesons are not changed. When we include an extra term in the lagrangian, $\mathcal{L}_\sigma$, to fit the experimental values for a_1 meson mass and width, there is not the same mixing in the vector and axial-vector correlator as shown from model independent calculation, and there is T^2 dependent correction to ρ meson mass. However, the effective masses of vector mesons are still not changed much

with temperature. The increase of the effective masses due to the pion loop corrections are almost canceled by the contribution from $\pi - a_1$ loop.

This result implies that the effect due to the modification of vector meson masses cannot be observed unless the temperature of the hadronic matter is very close to the critical temperature for the phase transition. Instead, at low temperature, there is an appreciable reduction in the coupling constant of the external vector current to vector mesons because of the mixing effect [12]. The reduction in the coupling constant leads to a suppression in the production rates of photons and dileptons from hot hadronic matter, which has stronger dependence on the temperature than the shift of the peak position in the spectrum [18].

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APPENDIX A: ρ-MESON SELF-ENERGY FROM ONE-LOOP DIAGRAMS

The expressions for the ρ-meson self-energy is given by

$$\Pi^{(a)}_{\mu\nu}(k_0,k) = -\frac{1}{2}g^2 T\sum\int \frac{d^3\vec{\mathrm{p}}}{(2\pi)^3}\frac{(2p_\mu - k_\mu)(2p_\nu - k_\nu)}{p^2(p-k)^2} \tag{A1}$$

$$\Pi^{(b)}_{\mu\nu}(k_0,k) = \left[g_{\mu\nu} - \frac{\lambda}{m_\rho^2}(k^2 g_{\mu\nu} - k_\mu k_\nu)\right] g^2 T\sum\int \frac{d^3\vec{\mathrm{p}}}{(2\pi)^3}\frac{1}{p^2} \tag{A2}$$

$$\begin{aligned}\Pi^{(c)}_{\mu\nu}(k_0,k) = {} & \frac{2}{f_\pi^2}T\sum\int \frac{d^3\vec{\mathrm{p}}}{(2\pi)^3}\frac{1}{p^2}\frac{1}{(p-k)^2 - m_a^2} \\ & \times\Big\{\eta_2^2 k^2(k^2 g_{\mu\nu} - k_\mu k_\nu) + 2\eta_2\bar{\eta}(k\cdot p)(k^2 g_{\mu\nu} - k_\mu k_\nu) \\ & +\bar{\eta}^2[(k\cdot p)^2 g_{\mu\nu} - (k\cdot p)(p_\mu k_\nu + p_\nu k_\mu) + k^2 p_\mu p_\nu] \\ & +\frac{\eta_1^2}{m_a^2}[(k^2)^2 p_\mu p_\nu - k^2(k\cdot p)(p_\mu k_\nu + p_\nu k_\mu) + (k\cdot p)^2 k_\mu k_\nu]\Big\}\end{aligned} \tag{A3}$$

where the superscript a,b,c denote the contributions from fig. 1-a,b,c, respectively, and the

$$\eta_1 = \frac{g^2 f_\pi^2}{2m_\rho^2}\left(\frac{1-\sigma}{1+\sigma}\right)^{1/2} + \frac{2g\xi}{\sqrt{1+\sigma}}\left(\frac{1-\sigma}{1+\sigma}\right)\frac{m_\rho^2}{m_a^2} \tag{A4}$$

$$\eta_2 = \frac{g^2 f_\pi^2}{2m_\rho^2}\left(\frac{1+\sigma}{1-\sigma}\right)^{1/2} - \frac{2\sigma}{\sqrt{1-\sigma^2}} \tag{A5}$$

$$\lambda = \frac{g^2 f_\pi^2}{2m_\rho^2}\left(\frac{1+\sigma}{1-\sigma}\right) - \frac{4\sigma}{1-\sigma}\left(1 - \frac{m_\rho^2}{g^2 f_\pi^2}\right) \tag{A6}$$

and $\bar{\eta} = \eta_1 - \eta_2$. When $\sigma = 0$, $2g\xi = 1$ and $Z^2 = 1/2$ we have $\eta_2 = 1/2$ and $\lambda = 1/2$.

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FIGURES

FIG. 1. One loop diagrams for the ρ-meson self-energy. The dotted, solid and double solid lines denote, respectively, the pion, ρ-meson and a_1-meson.

FIG. 2. Effective mass of ρ-meson at finite temperature. The dashed line is the result from the calculation with pions and rho-mesons. The solid line is the result obtained when a_1-mesons are included.